Rock strata heat in time-dependent underground heat modelling

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ABSTRACT

The paper presents a method to solve for the rock strata radial heat conduction across all the airways of a mine ventilation model. The method allows the investigation of a range of time-varying heat phenomena in underground mines. This contrasts with the practice of modelling the heat flow from rock strata using the Gibson function (1976), which gives the strata heat flow as a function of the temperature and velocity of the airstream, the age of the airway, and a range of rock thermal properties. The major drawback of the Gibson function is that it requires a fixed temperature and velocity over the life of the tunnel. Therefore, using it in modelling any situation requiring changes in these parameters (daily and seasonal atmospheric variation, fires, installation or shutdown of a cooling plant or fan) is problematic and achieved by introducing an often case-specific calibration to match to observed real-world results. Another problem exists in using the Gibson function on airways in isolation when they are interconnected. As heat flow to one airway reduces over time as the rock cools, changing airstream temperatures will affect the heat flow occurring in downstream airways, making the result of the function – in all but the smallest of mine ventilation networks – tend towards underestimating the historical temperature with respect to the actual temperature used in the function. The proposed method in this paper fully simulates the rock wall temperature radial distribution, allowing the heat history of the airway to be included in the heat flow calculated at any moment. From this, the paper provides a quantification of the error in the Gibson function for interconnected airways in a standard sized mine ventilation network. The paper also presents how this new method improves the accuracy of modelling of dynamic heat phenomena (such as seasonal atmospheric variation and fires), removing the need for calibration factors. Also demonstrated is how this enables transient modelling of heat over the mine life and ultimately better prediction of mine environment conditions and requirements for ventilation and cooling.

INTRODUCTION

A major factor in the heat and moisture in an underground mine is the geothermal heat that enters the airstream from the walls of the airways. While the underlying equations for this physical process are well understood, modelling it in an underground mine is difficult due to the three-dimensional layout of the airflow network. The heat transfer from the rock wall to the airstream can be modelled one-dimensionally per airway but modelling it over thousands of connected airways is more difficult. This paper seeks to present the current practice for handling geothermal heat in underground heat network modelling and explore the possibility of increasing the complexity of the modelling techniques and obtaining a more accurate and spatially and temporally resolved simulation.

When an airway is mined, the rock is at the virgin rock temperature (VRT). As soon as it is exposed to the air it begins to cool, with the rock closer to the air cooling first, and the cooling extending deeper into the rock over time. It cools by conduction of the heat from deep in the rock to the rock surface and then convective heat transfer from the rock surface to the airstream. If we simplify the airway to a cylinder the heat flux per square metre of exposed rock surface can be represented by:

$$q = k\left(\frac{\partial\theta}{\partial r}\right)_s = h(\theta_s - \theta_d)\ \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right]$$

where:

K	is the thermal conductivity of the rock [W/(m°C)]
θ	is the temperature [°C]
r	is the radial coordinate [m]
h	is the heat transfer coefficient for heat transfer from the rock surface to the air [W/ (m^2°C)]

θ_s	is the temperature of the rock at its surface, or at its interface with the air
θ_d	is the dry bulb temperature of the air (the forcing temperature), with the subscript $_s$ representing a value at the rock surface to air interface

But as heat is transferred between the air and the rock there is change over time of the rock wall temperature. The problem is simplified by assuming the heat conduction in the rock varies only in the radial direction, which allows the simplified radial heat conduction equation:

$$\alpha\left(\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}\right)=\frac{\partial\theta}{\partial t}\ \left[\frac{°\mathrm{C}}{\mathrm{s}}\right]$$

where,

α	is the rock thermal diffusivity [m^2/s]
t	is time [s] (Danko *et al,* 2012; McPherson, 1993)

Note, this equation is expressed differently in Danko *et al* (2012), but it simplifies to the same equation. With these two equations we can model the rock temperature variation into the wall. Normally we have a good idea of rock thermal conductivity and diffusivity and we know at t = 0 the rock is all at the VRT. The convective heat transfer coefficient, h, can be calculated as a function of the rock surface roughness and the air velocity across the rock surface.

One can solve the equations above numerically to obtain the heat flux at any moment for a given airway age, VRT, rock type, and air velocity and temperature. But it can be computationally expensive, so various short cuts and simplifications are used. The best known is the Gibson function (McPherson, 1993), which was presented as a quick way to obtain the heat flux for a given airway as a function of its age, rock thermal properties, air velocity and temperature. It allows the accurate calculation of heat flux with a handful of steps and is far less intensive than numerically solving all the heat variation into the rock. This is very useful for a network heat simulation which needs to have the heat flux from thousands or tens of thousands of airways. But the function has several simplifications, with the one of most interest being the assumption of a constant air temperature and speed.

In an underground mine environment, the air temperature and speed will not be constant. Temperature will vary with night and day and passing seasons (depending on proximity to the intakes). Also, an underground mine is a dynamic, constantly changing environment; sections of the mine might be opened then closed, the amount of vehicle heat sources might change, or refrigeration or heating units might be installed or replaced. None of these changes in the forcing air temperature of the rock heat transfer can be considered in the Gibson function.

To take daily and seasonal variation first; these effects are largely periodic, therefore their effect can be averaged out and the Gibson function should be able to calculate a good answer regardless. But it cannot tell us anything about those daily and seasonal variations, if they are of interest to us. For example the effect of the heat capacitance cannot be included, which will cause a phase lag in the temperature deeper underground with respect to the outside forcing temperature. Step changes in forcing temperature – from, for example, a refrigeration unit starting up or shutting down – cannot be modelled with the Gibson function assumption of constant temperature.

Another issue is the translation of this one airway method to a complex network heat simulation, where thousands of airways interact. Firstly, an airway downstream will have its forcing air temperature affected by the rock wall heat transfer of airways upstream, meaning in a hot mine, the Gibson function will generally underestimate the dry bulb temperature if used on multiple airways in a network.

The aim of this paper is to investigate these inaccuracies of the Gibson function and then to discuss the possibility of incorporating time dependent rock wall heat transfer into a network heat simulation. This will be computationally expensive, possibly prohibitively so. But doing so would not only allow the investigation of effects of changing forcing temperatures, but also allow better calibration of models with measured survey data. Presently, a heat simulation using the Gibson function simulates an average state, rather than the heat state at any given time; but this average state never exists.

Finding a way to simulate to a given moment, rather than to an average state would be useful for calibrating models with site temperature measurements.

METHOD

To investigate the effect of having the true air temperature history in an airway included in the rock strata heat flux, a numerical solver was written to model the radial heat distribution over time in a single airway. The single airway solver discretises the radial heat equation over the radial distance into the rock wall and then models the rock temperature deep in the wall as it changes over time in response to the air temperature and velocity.

Effort was put into making the solver run as quickly as possible, because it is anticipated that the solver would be run in parallel across hundreds or thousands of airways in a network model. The solver uses central difference finite differences to calculate the 1st and 2nd spatial derivatives in Equation 2, while the solution is stepped forward in time using the Runge-Kutta method, which proved the fastest method for a given accuracy. The radial temperature gradient reaches deeper into the wall over time; once the radial temperature gradient at the deepest point exceeds 0.01°C/m, the domain is extended.

To verify the solver works, we can run it for a constant temperature over the life of the airway and we should arrive near the same result as the Gibson function. For the calibration case shown below, the initial spatial discretisation at the rock wall is 0.01 m, and then increases into the rock wall as the spatial temperature gradients diminish. The time step is 150 secs. Increasing the spatial or temporal resolutions beyond these values produced only negligible further change. For this test, we will use the same airway parameters as used in Danko *et al* (2012). These are:

- Airway: width 3.5 m, height 5.7 m, perimeter, 18.4 m, area, 19.95 m^2.
- Average air temperature 28°C dry bulb, wet bulb 15.9°C, pressure, 87.15 kPa.
- Rock: VRT 28°C, specific heat 844 J/kg/K, thermal conductivity 3.15 W/m/K, density 2309 kg/m^3.

A forcing air temperature of 18°C is chosen. The development of the rock heat over time is shown in Figure 1, while the heat flux over time can be seen in the solid purple line in Figure 2a.

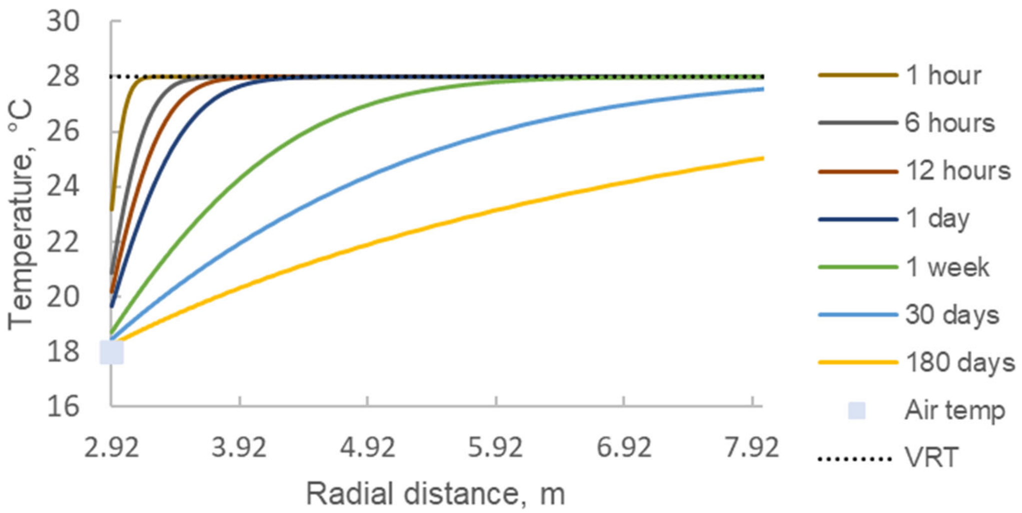

FIG 1 – Variation of rock temperature over time for an airway of hydraulic diameter 2.92 m, of VRT 28°C, exposed to constant air temperature of 18°C, with air flow quantity 50 m^3/s.

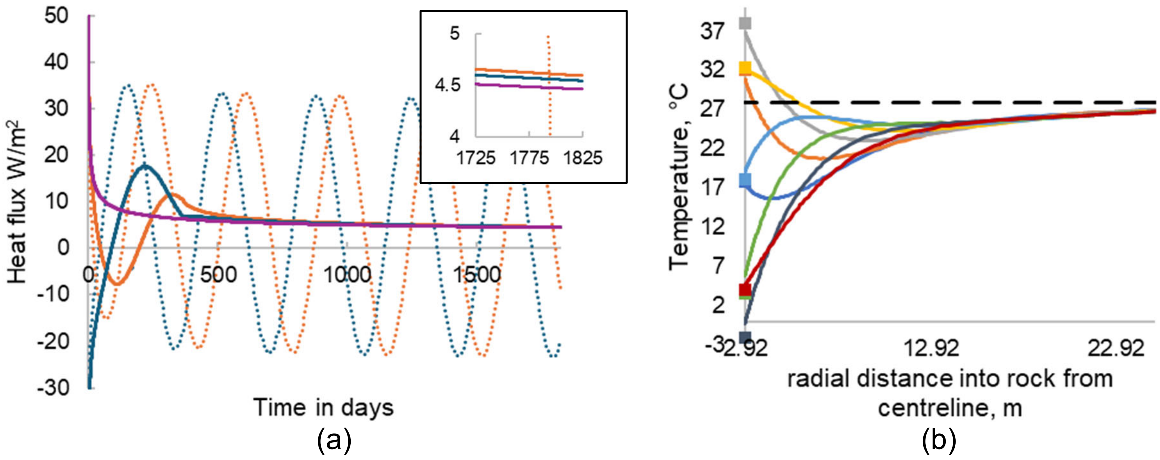

FIG 2 – For a single airway, in (a) the time dependent (dotted) and mean (solid) heat flux for three cases and in (b) the temperature inside the rock wall at eight times over one year for one particular case. More detail in text.

We can verify this solution by calculating the rock wall surface temperature and the heat flux from the Gibson function. The Gibson function for the same inputs returns a heat flux per unit of rock wall area of 7.32 W/m^2, which is within 2 per cent of the heat flux after 180 days for our solver, of 7.45 W/m^2; a similar accuracy is obtained extending out to five years. This is the stated accuracy of the Gibson function, so we can have confidence in the accuracy of the single airway solver presented here.

SINGLE AIRWAY

The advantage that the single airway solver has over the Gibson function is that we can now model variations over time in the input temperature.

Figure 2a plots in purple the heat flux per unit area over five years for the case with constant forcing temperature of 18°C; in dotted orange it shows the heat flux over five years for the same case as in Figure 1, but now with an annual sinusoidal temperature variation of 20°C added to the 18°C average temperature (giving maximum 38°C, minimum -2°C); the running one year average of this heat flux is plotted in the solid orange line; in the dotted and solid blue lines is shown the same case but with the annual variation shifted by three months. Effectively, the orange shows an airway mined in spring, blue an airway mined in the summer. Figure 2b shows the internal rock wall temperature variation at eight evenly spaced times over one year.

It is often thought that the linearity of the radial heat equation means that the annual variation when considering the average state can be ignored, but this is not quite true. On the graph we can see that the average heat flux for the constant forcing temperature is different to the averaged heat flux for the variable forcing. It is very different for the first year, and still different after five years (see the inset graph), but only slightly so; (note: for the Gibson function, this is possibly considered already in its stated accuracy of 2 per cent; the original reference for the function was unavailable to the author and this issue is not discussed in McPherson (1993). Another variable that affects the result is what time of the year the airway is first mined, with Figure 2 showing heat flux for a summer airway and a spring airway. These differences are due to the average cooling over time; the temperature difference between the rock and the air is what drives the heat transfer; what air temperature differences the rock wall is exposed to when it is early in its life are more critical than later in its life, because the rock is on average hotter when the airway is younger. So an airway opened in the summer will hit the colder part of the year earlier in its life than an airway opened in the spring, so it will lose more heat earlier, resulting in less heat flux later; the result of this can be seen in the higher heat flux for the spring airway after five years (see inset graph of Figure 2).

The Gibson function and time averaged result is significantly different from the running average heat flux for both cases until about one year of airway life. Of course the extent of this error would depend on the extent of the annual variation in temperature, and on the airway's exposure to this variation, which would be less the deeper in the mine the airway is.

MULTIPLE AIRWAY SOLVER

Another issue with the use of the Gibson function in network heat simulations is the interconnectedness of the many airways in a standard underground mine. An airway placed downstream, deep in the mine will be affected by the cooling/heating of airways upstream. This effect is not possible to include in a network heat simulation using the Gibson function. So in a hot mine, the Gibson function will tend towards underestimating the air temperature in downstream airways.

The multiple airway solver uses as a base the Ventsim DESIGN Heat Simulation tool, but with the Gibson-based rock strata heat transfer function replaced on every airway with the single airway solver described in the previous section. This will have a number of effects:

- such a solver will account for the effects of heating/cooling airways upstream on airways downstream
- it will be able to model the phase lag in underground temperature peaks with respect to atmospheric temperature peaks (due to the heat capacitance effect of the underground rock walls)
- it will be able to handle transient changes in forcing effects (such as seasonal changes, diurnal changes, and heating or refrigeration changes)
- and finally it will require far more computation.

For even a small mine, running the single airway solver over a few hundred airways for any useful period of time will require considerably more computer memory and operations than the existing standard heat simulations. This will be discussed more later.

To validate the multiple airway solver, we can refer to the work of Danko *et al* (2012). They simulated the varying temperature down a 6 km tunnel over a five year period. The tunnel uses the same parameters as described in the Method section of this paper. The temperature variation is 20°C around an average temperature of 28°C, so – in contrast to the results of Figure 2 which oscillated around 18°C – varying between 8°C and 48°C. Figure 3 plots the temperature variation 6 km into the tunnel over a five year period using the multiple airway solver.

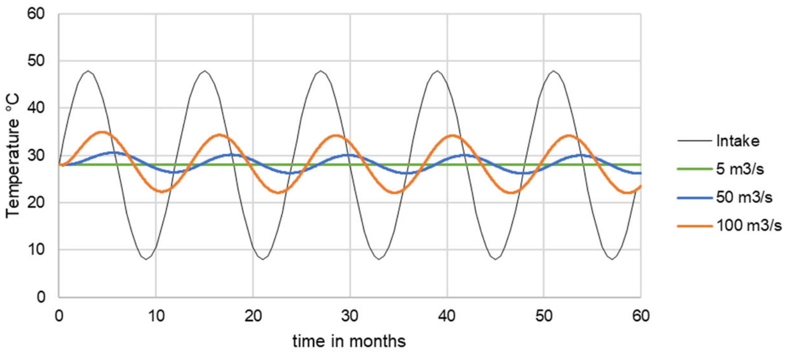

FIG 3 – The temperature variation at the 6 km mark for the test case at three different airflow rates, calculated from the multiple airway solver.

In the graph, the peaks of the temperature variation for the 50 m^3/s and 100 m^3/s cases lag the peaks of the intake temperature. There is also a reduction in the amplitude of the variation with respect to the intake temperature. This phase lag and amplitude reduction are the effects of the heat capacitance of the rock wall upstream of the point shown in the graph. We can measure the phase lag and amplitude decay as a function of the distance down the tunnel, allowing a direct comparison with figures 10 and 11 of Danko *et al* (2012).

Figure 4 plots the variation of amplitude decay and phase lag along the tunnel for the three cases shown in Figure 3. These plots compare well qualitatively to figures 10 and 11 of Danko *et al* (2012); but there is some difference in values. The amplitude decay in the three cases after 6 km is 0.0006, 0.100 and 0.309 for the 5 m^3/s, 50 m^3/s and 100 m^3/s cases respectively; while for the same values Danko *et al* (2012) report approximately 0, 0.07 and 0.23. For phase lag after 6 km, Figure 4 shows

8.98 months, 2.78 months and 1.563 months for the 5 m^3/s, 50 m^3/s and 100 m^3/s cases respectively; while for the same values Danko *et al* (2012) report approximately 8.2 months, 2.1 months and 1.1 months.

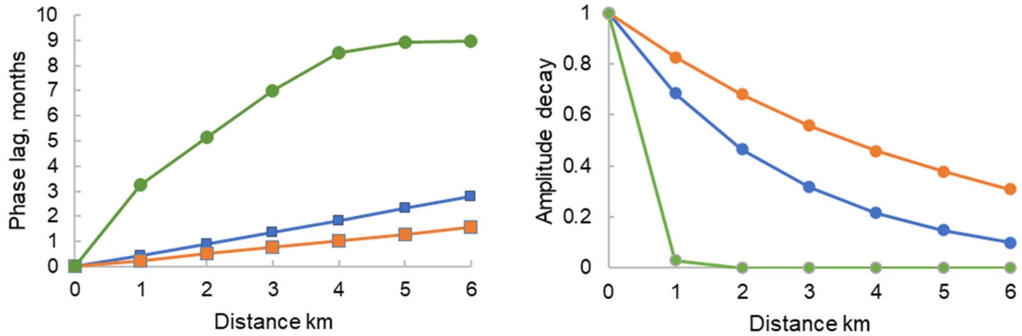

FIG 4 – (a) the variation in phase lag along the tunnel for the three airflow rates tested; (b) the variation of amplitude decay with respect to the intake amplitude for the same cases (green 5 m^3/s, blue 50 m^3/s, orange 100 m^3/s).

The match up in terms of change in phase lag and amplitude decay with distance is encouraging. The discrepancies in absolute values could be due to several factors. Firstly, Danko *et al* (2012) makes no mention of friction factor in their case, which should have an impact on the heat transfer coefficient for the rock wall to air interface (McPherson, 1993). In the current method an Atkinson friction factor of 0.012 kg/m^3 was used in the formulation of the heat transfer coefficient. A quick test with the current method shows doubling the Atkinson friction factor causes approximately 5 per cent and 2.5 per cent increases at 6 km in the phase lag and amplitude decay, respectively. secondly, the rock thermal diffusivity used in Danko *et al* (2012) is not consistent with the rock thermal conductivity, specific heat capacity and density presented, so there is some uncertainty. These potential differences in problem formulation between the two methods would likely create quantitative differences in result, without changing overall trends, which is what is seen in Figure 4.

Further cases are run to gauge the effect of more realistic conditions. In the 6 km tunnel shown above, all 6 km of the tunnel are modelled as present at time = 0. In reality, this 6 km tunnel would not entirely appear at one instant but be mined over time. It would also possibly be increasing in-depth, so an increase in VRT along the tunnel is likely.

Figure 5 plots the temperature variation at 6 km, along with the amplitude decay and phase lag for four cases where the VRT has been scaled up as if the tunnel is going deeper underground in the presence of a geothermal gradient.

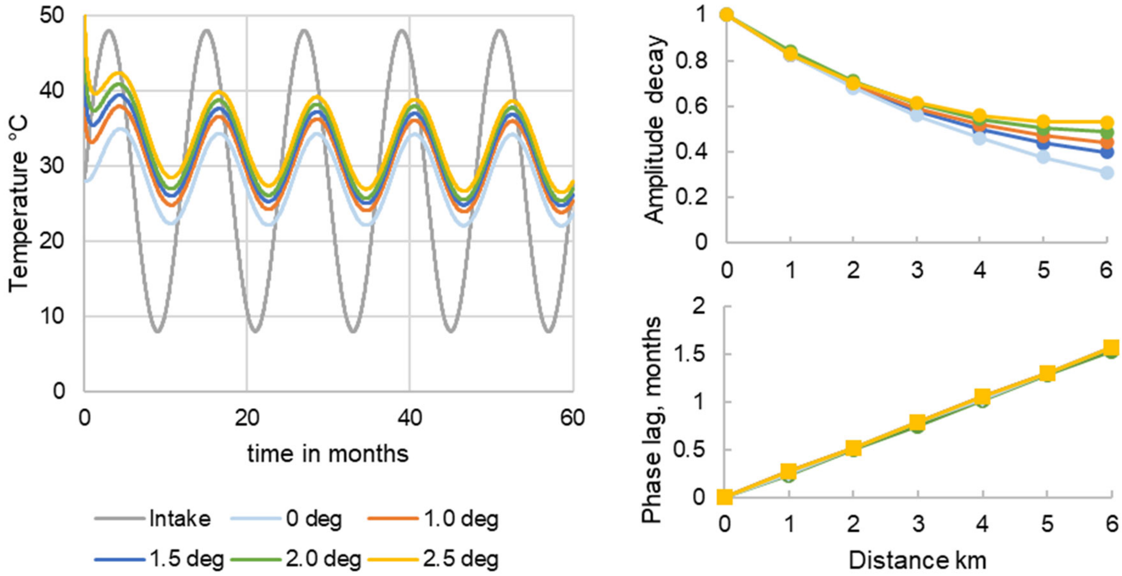

FIG 5 – Results for a flow of 100 m^3/s, varying the geothermal gradient along the tunnel. (a) shows the temperature variation at the 6 km mark, while (b) and (c) show the corresponding amplitude decay and phase lag variation.

Each airway was changed from VRT = 28°C to a value corresponding with its distance from the intake and the geothermal gradient and an assumed decline gradient of 15 per cent. Note, the elevation of the airways was not changed, so this case does not include any effect from changes in air density resulting from changes in-depth along the decline. The expected higher temperatures can be seen in the temperature variation at 6 km. The phase lag is not affected by the higher rock temperatures, while the amplitude of the heat variations is sustained further downstream. These results are consistent with there being simply more heat energy available in the rock mass, thus sustaining temperatures. It seems the phase lag remains independent of the amount of heat energy available, instead having a dependence on the speed at which heat energy in the air is carried down the tunnel, as shown in Figure 4.

A further complication comes from the tunnel not instantly appearing at a moment in time, but rather being constructed over a set amount of time. More tests were run whereby a mining date was assigned to each airway, so that airways only took part in the simulation once the running time of the simulation had passed the date mined. Adding this factor had no effect on the phase lag or the amplitude decay. The only effect was to delay the start of the cooling process on each airway, but since in this case downstream airways have no effect on upstream airways, there was no interesting change in results beyond this.

The final case shown here features the same tunnel but with a cooling power introduced at the intake at the two year mark, and then removed at the four year mark. Such an example demonstrates how such a simulation tool could be used to measure transient effects of large changes in temperature. In this example, a geothermal gradient of 2.0°C is applied at a supposed 15 per cent decline gradient (VRT varying from 28°C to 46°C). The cooling unit, when applied, is modelled by setting a maximum intake wet bulb temperature of 20°C, and if adjusted, also setting the dry bulb temperature to 20°C. Figure 6 plots the resultant temperature variation at the 6 km mark.

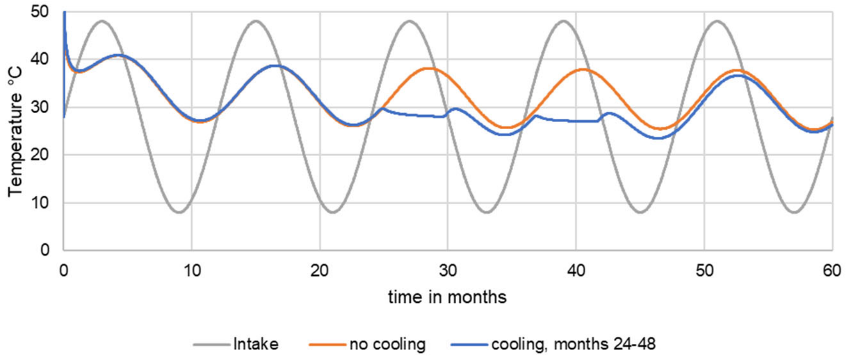

FIG 6 – The temperature variation at the 6 km mark for a case with geothermal gradient 2.0°C/100 m, flow quantity 100 m^3/s, and with a cooling unit limiting the intake temperature to a maximum of 20°C wet bulb temperature, but only operating between months 24 and 48.

The effect of the added cooling can be seen in the third and fourth years, and then the lingering effect from the rock heat capacitance seen in the small difference in the fifth year. Contrast such a result with a steady state simulation which uses the Gibson function with an airway age set at five years. In such a case, a constant forcing temperature must be chosen. Firstly, without the cooling power, if the average forcing temperature of 28°C is chosen, this results in a temperature at the 6 km mark of 31.2°C. If the oft-used method of picking a forcing temperature at the 95 per cent percentile of maximum temperatures is used, the temperature at the 6 km mark returned is 42.5°C. If the cooling unit is added, the temperature returned at the 6 km mark is 31.2°C.

These values returned from the steady state heat simulation are useful in determining the general effect and the relative difference with other simulation results, but do not provide nearly as much detail and resolution of information as is available with a simulation performed with the multiple airway solver. With the multiple airway solver, it is no longer reporting time-averaged values of temperature, but rather temperatures at specific times. Time-specific measurements of temperature are difficult to correlate with a simulated time-averaged temperature from a method using the Gibson function but could be matched with a time-resolved temperature from the multiple airway solver, such as the temperatures shown in Figure 6.

Further to calibrating model heat data, such a solver can be used to predict more accurately temperatures deeper underground which will have significant lag or amplitude decay from intake temperature variations. This currently is not accounted for in heat simulations using the Gibson function. Also not accounted for are abrupt changes in forcing temperatures resulting from operational changes in the mine. These abrupt changes could result from: the installation of a cooling or heating plant; a switch from vehicle haulage to a hoisting shaft; the opening up then shutting down of development levels; or a fire.

In underground fire ventilation modelling, simplifications are made in the modelling to account for the absorption by the rock mass of the massive amount of heat coming from the fire. This is an important component of the fire modelling task. The Ventsim VentFIRE method (Brake, 2013) makes an assumption about the thickness of rock wall that heats up in response to the fire, a thickness which is calibrated for the time frame of a fire model, usually about 1 hr. Other fire simulation methods simply take off a fixed factor of the fire heat to account for tunnel heat absorption. Another example of such *ad hoc* methods is in the flywheel solver of Griffith and Stewart (2019) which is calibrated to account for observed phase lag and amplitude decays for seasonal temperature variations. All these methods are calibrated to specific time scales and, to greater or lesser extent, rock thermal properties and are inflexible to varying degrees to changes in total simulation time and rock parameters. The multiple airway solver would perform this task more accurately and with greater flexibility, at all-time scales and across different rock types.

There are however significant drawbacks to using the multiple airway rock heat solver. The amount of computational memory and computing time required is orders of magnitude greater than what mine ventilation modellers are accustomed to. Most of the results in this study were obtained on an airway network consisting of 60 airways. Running the solver for five years on this network using the author's computer required around 10 mins of computation time; typical mine models consist of 500 to 20 000 airways, often more, and the computation time will scale roughly proportionally with the number of airways. However, there remains large scope to improve the speed of the solver; work on speeding up the time-stepping algorithm of the single airway solver was stopped with the intention of obtaining the results presented, so there is still work to do on optimising. There is also the possibility in the future of using analytical solutions of the radial heat equation, rather than the numerical one currently used, that should be faster (Hefni *et al*, 2022). One more thing to consider is the possibility of the engineer adjusting workflows to longer computation times; standard simulation times vary widely across engineering disciplines, with mine ventilation sitting at the extreme shorter end.

Finally, this study has avoided discussing moisture transfer from the rock to the air as it would constitute a complete further study unto itself, and all the results reported feature entirely dry heat transfer at the rock-air interface. There are ad hoc methods for handling moisture (such as in McPherson, 1993) which are not strictly correct and usually modelling with moisture requires after the fact adjustment of wetness fractions or moisture flows to account for the extra heat flow resulting from the presence of the moisture and the latent to sensible heat flow ratio. Incorporating moisture into the solver presented in this study without resorting to generalised moisture fractions or breakups between sensible and latent heat would be a significant challenge and an interesting avenue of further investigation.

CONCLUSIONS

The study has presented a 1D radial heat solver and its calibration against existing methods. The single airway solver has been incorporated into an existing heat network solver to produce time-resolved heat simulations of heat flows along a tunnel. The results have shown agreement with another study in the literature. Further results presented showed the effects of changes to rock parameters and temperature forcing, demonstrating the potential use and high versatility of such a solver to the mine modeller. Finally, the various challenges lying ahead for using such a solver on a standard mine model were discussed.

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